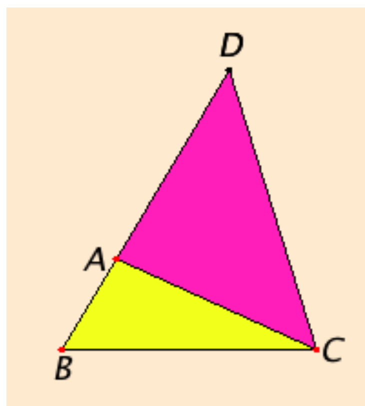
Homework for MCB3

Due October 24

To really do what I want to do with you guys next, we have to get through a bunch of Euclid. That's why this homework is entirely propositions.

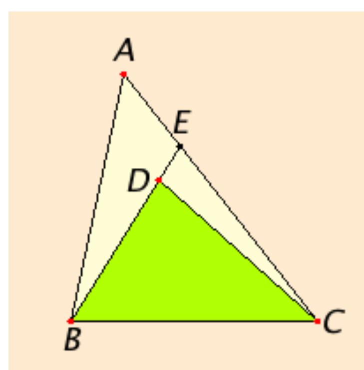
1 Proposition 20

Prove what your intuition tells you: that two sides of a triangle must add to more than the third side. The diagram should get you started. Start with $\triangle ABC$ and extend $\overline{AB}$ in such a way that $\overline{AD} \cong \overline{AC}$. See which angles are bigger than which others (hint: check for isosceles triangles) and use theorems we have proven in class.



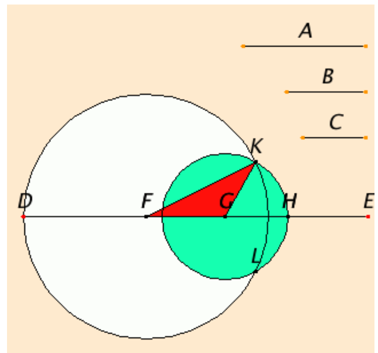
2 Proposition 21

Prove that if you draw a triangle inside another triangle (but sharing two of its vertices, and therefore one side), then the other two sides of the new triangle will be shorter (more precisely, their sum will be smaller), and the angle at which they meet will be larger. The diagram should get you started. Use Proposition 20 on $\triangle AEB$ and $\triangle CED$ to get the part about lengths. Then use the exterior angle theorem we proved the previous week with the same triangles to get the part about the angle.



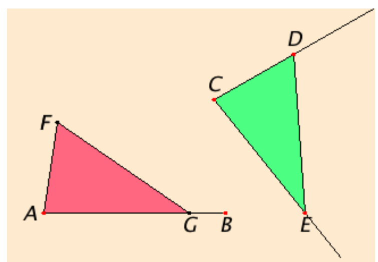
3 Proposition 22

Take three segments of random lengths (but in accordance with Proposition 20!) and construct a triangle from them. The diagram should get you started. (Kirill: we didn't have time to review how to move a segment in a construction; you may ask your father, and you may mention to him that it's Euclid's Proposition 2.)



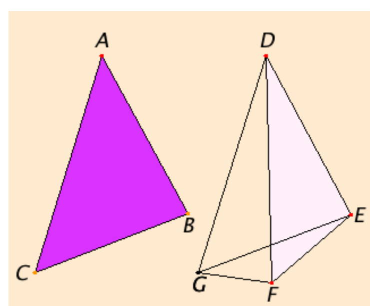
4 Proposition 23

Find a way to "move" an angle like what we already have for moving line segments. The diagram should get you started. Start with $\angle C$ and $\overline{AB}$. Choose D and E at random on the sides of $\angle C$ and then use Proposition 22 to move the triangle.



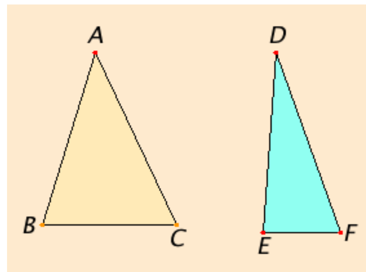
5 Proposition 24

Prove that if two triangles have two corresponding congruent sides, but in one of them, the angle between them is greater than in the other, then the third side will be bigger in the triangle with the bigger angle. The diagram should get you started. Start with $\overline{AC} \cong \overline{DF}$ and $\overline{AB} \cong \overline{DE}$ with $|\angle CAB| > |\angle FDE|$; then construct G so that $\overline{DG} \cong \overline{DF}$ and $\angle GDE \cong \angle CAB$. Then pay attention to $\triangle GFE$ and think about angles and sides.



6 Proposition 25

Prove the converse of Proposition 24. Start with $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$, with $|\overline{BC}| > |\overline{EF}|$, and do a reductio ad absurdum. Try $\angle BAC \cong \angle EDF$ and $|\angle BAC| < |\angle EDF|$.

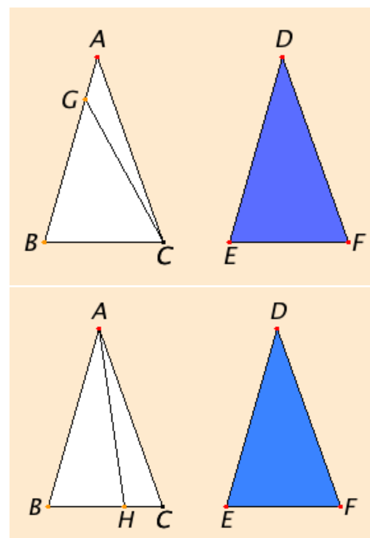


7 Proposition 26

We have proven the "side-side-side" and "side-angle-side" theorems of triangle congruency. We have also demonstrated that "angle-side-side" does not work as a theorem of congruency. Now it is time to try "angle-side-angle" and "angle-angle-side."

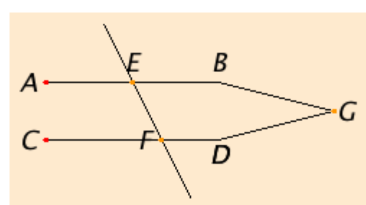
For ASA, start with $\angle ABC \cong \angle DEF$, $\angle BCA \cong \angle EFD$ and $\overline{BC} \cong \overline{EF}$. Do the classic reductio ad absurdum where you suppose $\overline{AB} \ncong \overline{DE}$, and let $|\overline{AB}| > |\overline{DE}|$, then place G such that $\overline{BG} \cong \overline{DE}$ and follow the logical chain.

For AAS, start with $\angle ABC \cong \angle DEF$, $\angle BCA \cong \angle EFD$ and $\overline{AB} \cong \overline{DE}$. Then suppose $|\overline{BC}| > |\overline{EF}|$ and place H so that $\overline{BH} \cong \overline{EF}$. Same reasoning.



8 Proposition 27

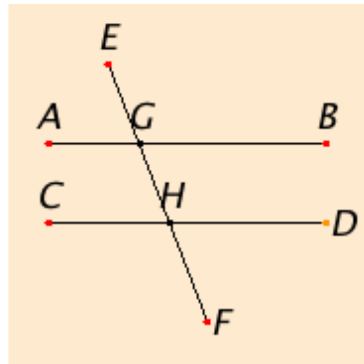
Prove that if a transversal[a] cuts across two lines forming congruent alternate interior angles, then the lines are parallel. Start with $\angle AEF \cong \angle EFD$ and do a reductio ad absurdum, assuming $\overline{AB} \nparallel \overline{CD}$; in which case they can be extended to intersect at G. Then take a look at $\triangle GEF$ and its exterior angles. Remember our theorem about those?



[a] A *transversal* is a non-parallel line.

9 Proposition 28

Two lines are intersected by a transversal. Prove that if the exterior angle is congruent to the interior and opposite angle on the same side (of the transversal), then the lines are parallel. Prove that if the two interior angles on the same side are supplementary, then the lines are parallel. Start with $\angle EGB \cong \angle GHD$, use the vertical angle theorem we once proved together, and then Proposition 27. Then start with $\angle BGH$ being supplementary to $\angle GHD$; check what other angle pairs are supplementary and use Proposition 27.



10 Proposition 29

Prove all the converses for Propositions 27 and 28. Use the illustration for Proposition 28, but now start by taking $\overline{AB} \parallel \overline{CD}$. As we have done basically every time with the converse, use reductio ad absurdum. Suppose $|\angle AGH| > |\angle GHD|$. Add $|\angle BGH|$ to both sides and then use geometric reasoning. You may use the fact that the contrapositive[a] of a true statement is always true (and the contrapositive of a false statement is always false).
Also, consider that $\angle AGH$ is vertical to $\angle EGB$, form an equation, add $|\angle BGH|$ to both sides. At this point, all the converses should be proven.

[a] The *contrapositive* of "if a then b" is "if not b, then not a."